Short Note: ELECTRIAL, ELECTRONICS & COMPUTER SCIENCE

Modeling and controller design of cart inverted pendulum system using MRAC scheme

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Abstract: The Cart-Inverted Pendulum System (CIPS) is a classical benchmark control problem. The control of this system is challenging as it is highly unstable, highly non-linear, non-minimum phase system and under actuated. The basic control objective of the inverted pendulum is to maintain the unstable equilibrium position, by controlling the force applied to the mobile cart in the horizontal direction. In this paper a controller is designed for the stabilization purpose based on the stability theory of Lyapunov which is one of the model reference adaptive control (MRAC) methods. The performance of the proposed control algorithm is evaluated and shown by means of digital simulation.

Keywords: Lyapunov, MIT rule, Model Reference Adaptive Control, Cart Inverted Pendulum

1. INTRODUCTION

The International Federation of Automatic Control (IFAC) Theory Committee in the year 1990 has determined a set of practical design problems that are helpful in comparing new and existing control methods and tools as benchmark control problems. An Example of such a classical benchmark control problem is a Cart-Inverted Pendulum System (CIPS). Its dynamics resembles with that of many real world systems of interest like missile launchers, pendubots, human walking and segways and many more. The control of this system is challenging as it is highly unstable, highly non-linear, non-minimum phase system and under actuated [1].

2.

The system consists of a rigid mass connected to a cart, which is constrained to move along a horizontal direction.The inverted pendulum is balanced [10] by controlling the pendulum's angle beneath the carts position..A force is applied to this cart: if appropriate forces are applied the pole can be kept in various positions from falling over. It is obvious

that under actuated systems have several advantages, so the control theory of under actuated systems is analyzed and investigated perceptively [3].

Several control algorithms are already implemented in the field of stabilization of inverted pendulum. It is well known that proportional integral derivative (PID) controllers [4]are widely used in control systems. The design of these controllers, however, are generally carried out using some tuning approaches such as Ziegler-Nichols method, which may not ensure good looprobustness, and it is very difficult to meet the design constraints [5].Sliding mode control, which is widely used for control of under actuated systems, have the potential problem of chattering, which is a high frequency oscillation present during the control [6].A neural network based motion control of cart inverted pendulum system is proposed in [7].he controller is developed for wheeled inverted pendulum models like SEGWAY, which is an example of inverted pendulum system. The application of model reference scheme and Linear quadratic regulation is also investigated in this work. Using this indirect control trajectory (the desired trajectory of forward movement) developed; the controller was able to indirectly control the tilt angle such that it tracks the desired trajectory asymptotically. The developed scheme achieves dynamic balance and desired motion tracking.

In this paper model reference adaptive control algorithm based controller is used to stabilize the pendulum. In MRAC,[2] the technical demands and the desired input output behavior of the closed-loop system are given via the corresponding dynamic of the reference model. Therefore, the basic task is to design such a control, which will ensure the minimal error between the reference model and the plant outputs despite the uncertainties or variations in the plant parameters and working conditions. The Main approach is to design a model reference adaptive controller using the stability

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theory of Lyapunov. This theory assures that the system at equilibrium point is asymptotically stable and is preferred for a second order system[8][9] as it yield better performance than MIT Rule.

2. MODEL REFERENCE ADAPTIVE CONTROL

When the plant parameters and the disturbance are varying slowly, or slower than the dynamic behavior of the plant, then a MRAC control scheme can be used. In this scheme desired specifications are given in the form of reference model. The MRAC structure consists of four main parts: the plant, the controller, the reference model and the adjustment mechanism. The schematic diagram of such system is shown in Fig 1. Basically it consist of two loops, first is for normal feedback control and second loop for controller parameter adjustment. The reference model tells how the process output should give response to the command signal. The output of reference model and plant is compared and error between them is given as a feedback through parameter adjustment loop. The parameters of the controller are updated such as to minimize
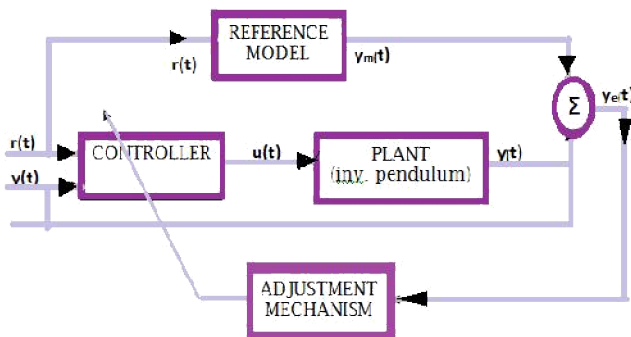

Fig. 1. Model Reference Adaptive Control

the error till it becomes zero. There are mainly two approaches to implement the MRAC, namely, MIT rule and Lyapunov theory. The drawback of MIT rule based MRAC design is that there is no guarantee that the resulting closed loop system will be stable. To overcome this difficulty, the Lyapunov theory based MRAC can be designed, which ensures that the resulting closed loop system is stable. The Lyapunov theorem states that If there exists a function $V: R^n \rightarrow R$ that is Positive definite such that its derivative along the trajectories is neagtive semi definite then the system is said to be stable,and the function V is known as the Lyapunov function.

$$\frac{dV}{dt} = \frac{\partial V^T}{\partial x}\frac{dx}{dt} = \frac{\partial V^T}{\partial x} f(x) = W(x) \tag{1}$$

3. MODELING OF INVERTED PENDULUM SYSTEM

In the Cart-pole system the pendulum rod is free to oscillate around a fixed pivot point attached to a cart which is controlled by a motor and is constrained to move in the horizontal direction. When the rod is placed in the upright vertical position, it is in an unstable equilibrium point. The objective of the controller is to apply a force to move the cart so that the pendulum remains in the vertical unstable position. The cart pole system is shown in Fig 2, where is the angle of
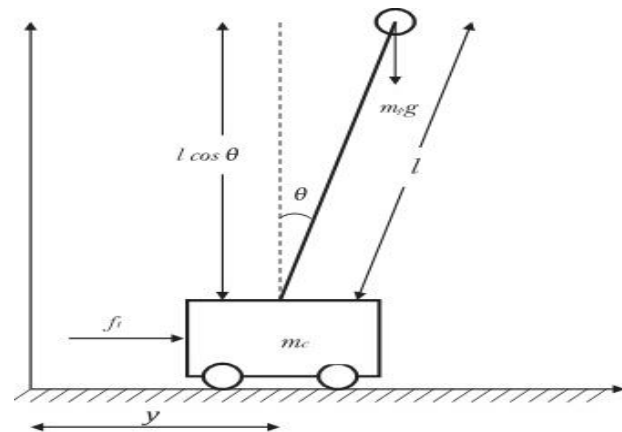

Fig. 2. Model Of A Cart Pole System

pendulum, x is the displacement of the cart and F is the control force applied, parallel to the rail, to the cart. It may seem that the inverted pendulum balance can be achieved by controlling the angle of the pendulum.

4. DESIGN OF MRAC CONTROLLER

The MRAC structure consists of four main parts: the plant, the controller, the reference model and the adjustment mechanism. The reference model is chosen to obtain the desired course for the plant output to follow. A standard second order differential equation was chosen as the reference model and the second order linear model of the plant was determined.

5. RESULTS AND DISCUSSION

The model of inverted pendulum with MRAC controller and its system parameters are shown in Fig. 3 and Table 1. When the system is subjected to an external distrubance the controller effectively balance the pendulum by stabilizing the angle of the pendulum Figure 4 and by controlling the cart position Figure 5 by genearting necessary control outputs. The

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error signal generated to adjust the adaptive law mechanism is shown in Figure 6.

TABLE I
SYSTEM PARAMETERS OF CART POLE PENDULUM

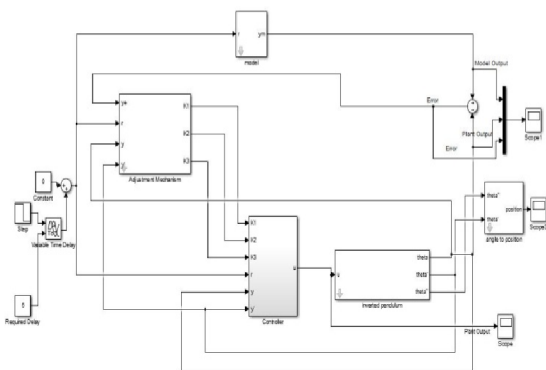| Parameter	Value
g-gravity	9.81m/s^2
l-length of pole	0.38m
M-cart mass	2.4Kg
m-pole mass	0.23Kg
J-moment of interia	0.009Kg-m^2
b-cart friction coefficient	0.05Ns/m
d-damping coefficient	0.005Nms/rad

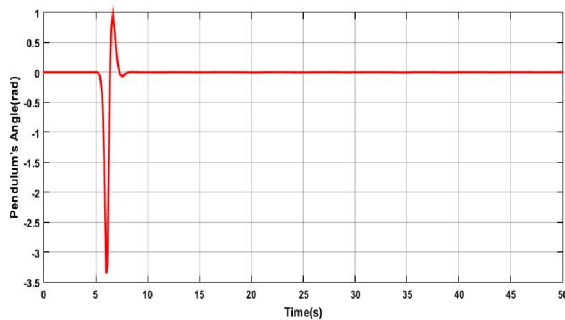Fig. 3. Simulink model of cart inverted pendulum system with controller
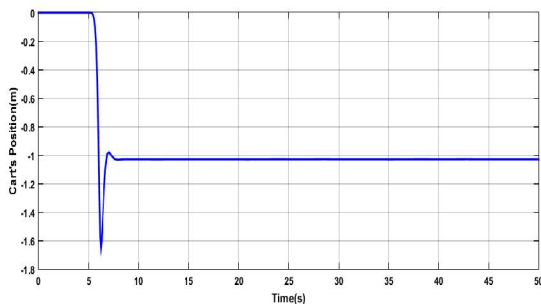

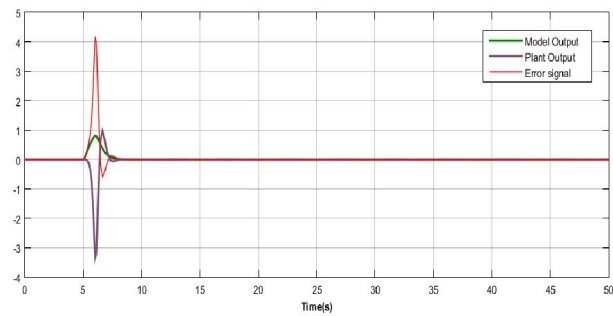Fig 4. Angle of the Pendulum

Fig 5. Position of the Cart

Fig 6. Adaptive Error Signal

In the normal case when there is no controller the pendulum angle will be settled at rad,ie, in the stable equilibrium point. But the developed controller effectively balances the pendulum at the unstable equilibrium point. It can also be observed from the simulation results that the cart reaches the mean position after balancing the pendulum. This indicates that controller developed helps to balance the cart and also generates necessary control output in order to move the cart into its stable position.

6. CONCLUSION

The cart-pendulum system is an under actuated nonlinear system. Systems having less number of actuators than the degrees of freedom available are known as under actuated systems. They are of high interest since their cost and complexity are very low. Study of under actuated systems gave insight into the structure and dynamics of higher order systems like underwater vehicles and space ships etc. The inverted pendulum cannot be balanced in the upright position by the direct control of the angle . The angle is virtually controlled by the movement of the cart in appropriate way. The cart-pendulum system can be linearized around its unstable equilibrium point in order to derive the controller. The Model Reference adaptive control based controller is proposed in this project. When the plant parameters and the disturbance are varying slowly, or slower than the dynamic behavior of the plant, then a MRAC control scheme can be used.

MRAC controller is efficient in controlling applications where mismatched uncertainties are present. It is also efficient in keeping the position tracking error very low.This technique involve a number of algebraic process which involves the stabilization of systems using Lyapunov Functions. The controller ensured the stability of the cart-inverted pendulum system in the upright position also it ensured the stability of

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the internal dynamics of the system, i.e. it considered controlling the cart motion on the rail.

The dynamic model of the pendulum system is linearized in this work. The ideas derived from the linearized controller can be used for developing the nonlinear controller for swing up and stabilization purposes. The most important point is how to select an appropriate Lyapunov Function for MRAC design of the nonlinear controllers to balance and stabilize the pendulum like systems in design process. It is clear that the linear controller only can achieve our control goals in the linearized stabilization zone.

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